ILLINOIS CERTIFICATION TESTING SYSTEM

FIELD 115: MATHEMATICS

TEST FRAMEWORK

November 2003

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Processes and Applications
Number Sense and Measurement
Algebraic Patterns, Symbols, Functions, and Models
Geometric Methods
Probability and Statistics

SUBAREA I—PROCESSES AND APPLICATIONS

0001 Understand how to communicate mathematical content and concepts, and develop and utilize a variety of problem-solving techniques.

For example:

- Demonstrate an understanding of learning styles and learning strategies in mathematics.
- Recognize how to create effective learning environments in which mathematics students work collaboratively in one-on-one, small-group, and large-group contexts.
- Demonstrate an understanding of strategies for teaching reading in the content area of mathematics.
- Demonstrate knowledge of how to communicate verbally and in written, visual, and symbolic forms using appropriate technology.
- Demonstrate an understanding of problem-solving strategies (e.g., using a diagram, making an organized list, working backwards using logical reasoning, making a simpler problem, acting out or using objects, systematic guessing and checking, looking for patterns, estimating, eliminating extraneous information).
- Recognize how to use problem explorations and modeling to extend the mathematical knowledge of all students.
- Generalize the results of problems and extend them to other problem situations.

0002 Understand how to apply appropriate reasoning techniques to concepts, procedures, and conjectures, and make connections with and among the various branches of mathematics and other disciplines.

For example:

- Recognize valid justifications for the application of concepts, procedures, and theorems in a given situation.
- Distinguish between inductive and deductive reasoning.
- Demonstrate an understanding of how to develop conjectures and evaluate their validity.
- Identify and apply connections within the mathematics curriculum (e.g., geometric substantiations of algebraic formulae such as demonstrating the connection of the distance formula in coordinate geometry via the Pythagorean Theorem, geometric interpretation of the integral, applications of matrices to geometric figures).
- Demonstrate knowledge of mathematical connections to other disciplines (e.g., rate of change as applied to business, economics, physics, chemistry, biology).
- Demonstrate knowledge of the historical development of mathematics, including contributions of men and women from various cultures.

0003 Understand how to select, integrate, and use appropriate technologies.

For example:

- Demonstrate an understanding of the capabilities and benefits of current and emerging technologies in the content area of mathematics.
- Select appropriate technology (e.g., spreadsheet, graphing calculator, manipulatives) to solve problems throughout the mathematics curriculum.
- Demonstrate an understanding of how to select and integrate appropriate technology for mathematics instruction.
- Demonstrate knowledge of CAS (computer algebra systems).
- Develop and apply algorithms to solve problems using technology.

SUBAREA II—NUMBER SENSE AND MEASUREMENT

0004 Understand the concepts of number, number theory, and numeration systems.

For example:

- Demonstrate an understanding of place value, order, magnitude, absolute value, and estimation.
- Represent and operate upon numbers using a variety of methods.
- Perform operations in any number base and convert between different number bases.
- Solve problems involving prime and composite numbers, least common multiples, modular arithmetic, and greatest common factors.
- Solve problems involving integers, fractions, decimals, percents, and ratios and proportions.

0005 Understand properties of the real and complex number systems as they apply to algorithms of operations.

For example:

- Demonstrate an understanding of real number properties and operations and how they apply to algorithms and algebraic expressions.
- Use polar and vector representations of complex numbers.
- Compute and interpret the results of computations using complex numbers and matrices.
- Demonstrate an understanding of iterative (e.g., infinite decimals, fractals) processes as they relate to fractals and other applications.
- Use numeric approximations as a basis for numeric integration and numeric-based proofs.

0006 Understand customary, metric, and nonstandard measurement.

For example:

- Demonstrate knowledge of standard, nonstandard, and emerging units (e.g., graphic screen pixels, font size) of measurement.
- Apply attributes of length, area, volume, capacity, time, temperature, angles (degree and radian measure), weight, and mass to solve problems.
- Solve problems using derived measures (e.g., density, work, velocity), conversion factors, and dimensional analysis.
- Use nonlinear measuring scales (e.g., Richter, decibel, pH) to solve practical problems.
- Determine acceptable measures of accuracy and calculate relative error in a given situation.

0007 Understand procedures for computing or estimating measures of multidimensional objects.

For example:

- Read and interpret scale drawings, topographical maps, and architectural drawings.
- Explain how changing one measure of a multidimensional object may affect other measures.
- Use trigonometric ratios to solve a variety of problems.
- Apply measurement formulas to irregular shapes, regions, and solids.
- Solve problems involving indirect measurement.
- Use modeling and visualization to predict, estimate, and determine measurements.

SUBAREA III—ALGEBRAIC PATTERNS, SYMBOLS, FUNCTIONS, AND MODELS

0008 Understand concepts, representations, and relationships of variables and patterns.

For example:

- Represent mathematical situations symbolically, numerically, and graphically.
- Identify, complete, and extend patterns, sequences, and series and analyze their properties.
- Use recursion and the principle of mathematical induction to solve problems.
- Translate between word situations and algebraic sentences.
- Apply properties of real numbers in algebraic contexts to manipulate and simplify algebraic expressions and solve equations.
- Use the properties of relations and functions (e.g., domain, range) and their symbolic, numeric, graphic, and verbal representations.
- Use a formal axiomatic system to construct and analyze proofs.
- Demonstrate an understanding of group structures and their application to symmetry.
- Demonstrate an understanding of rings and fields and their relation to algebraic properties of real numbers.

0009 Understand and apply concepts and representations of linear relations and functions.

For example:

- Represent linear relations and functions in symbolic, numeric, graphic, and verbal forms.
- Recognize and apply slopes and intercepts to construct, analyze, and interpret linear equations and inequalities and their graphs.
- Apply linear relations, functions, and systems to model and solve a variety of problems.
- Represent and solve systems of linear equations and inequalities graphically and algebraically, including matrix methods.
- Apply principles and properties of linear algebra (e.g., vectors, matrix algebra, vector spaces) to solve problems.

0010 Understand and apply concepts and representations of quadratic relations (including conic sections) and functions.

For example:

- Represent quadratic functions and relations in symbolic, numeric, graphic, and verbal forms.
- Recognize and apply properties of symmetry, roots, intercepts, and vertices to construct, analyze, and interpret quadratic relations and their graphs.
- Recognize and apply the properties of hyperbolas, parabolas, circles, and ellipses to model and solve problems.
- Solve systems of quadratic equations and inequalities graphically and algebraically.
- Apply quadratic relations, functions, and systems to model and solve a variety of problems.

0011 Understand and apply concepts and representations of polynomial, absolute value, radical, and rational functions and inequalities.

For example:

- Represent polynomial, absolute value, radical, and rational functions and relations in symbolic, numeric, graphic, and verbal forms.
- Recognize and apply symmetry, roots, intercepts, critical points, asymptotes, and vertices to construct, analyze, and interpret polynomial, absolute value, radical, and rational functions and inequalities and their graphs.
- Recognize and apply the properties of polynomial, absolute value, radical, and rational functions and equations to solve problems.
- Solve systems of polynomial, absolute value, radical, and rational equations and inequalities graphically, algebraically, and numerically.
- Apply polynomial, absolute value, radical, and rational relations, functions, and systems to model and solve a variety of problems.
- Recognize and apply the algebraic properties of polynomial and rational functions (e.g., factoring, partial fractions).

0012 Understand and apply concepts and representations of exponential, logarithmic, and trigonometric functions.

For example:

- Represent exponential, logarithmic, and trigonometric functions in symbolic, numeric, graphic, and verbal forms.
- Recognize and apply properties of symmetry, roots, intercepts, critical points, and asymptotes to construct, analyze, and interpret exponential, logarithmic, and trigonometric functions and inequalities and their graphs.
- Apply the equations of exponential, logarithmic, and trigonometric functions and systems to model and solve a variety of problems (e.g., periodic motion, compound interest, exponential growth and decay).
- Solve systems of exponential, logarithmic, and trigonometric equations and inequalities graphically and algebraically.

0013 Understand the historical development and applications of calculus.

For example:

- Demonstrate knowledge of the history of mathematics leading up to calculus (e.g., slope of tangent line as rate of change, using geometric methods to determine the area under a curve).
- Apply the concept of a limit to analyze properties of functions (e.g., continuity, asymptotes) and series.
- Apply principles of differential calculus to solve a variety of problems (e.g., rates of change, optimization, analyzing functions).
- Apply principles of integral calculus to solve a variety of problems (e.g., finding areas and volumes, describing the motion of an object).
- Represent limits, derivatives, and integrals symbolically, numerically, graphically, and verbally.

SUBAREA IV—GEOMETRIC METHODS

0014 Understand properties of points, lines, planes, and space and their relationship to Euclidean and non-Euclidean geometry.

For example:

- Demonstrate an understanding of points, lines, planes, and space and their geometric applications.
- Apply definitions, axioms, and theorems of Euclidean geometry to develop different types of proofs (e.g., direct, indirect, flow, paragraph).
- Solve a variety of problems in Euclidean geometry (e.g., justify geometric constructions).
- Use the formal axiomatic system of geometry to construct and analyze proofs.
- Compare and contrast the structures of Euclidean and non-Euclidean geometries (e.g., hyperbolic, elliptic).

0015 Understand properties of two- and three-dimensional shapes.

For example:

- Apply characteristics of two- and three-dimensional figures to describe, analyze, and categorize two- and three-dimensional figures.
- Apply the principle of congruence to explore properties of geometric figures and prove theorems.
- Apply concepts of similarity and congruence to analyze the properties and compare the measures (e.g., perimeter, area, volume) of two- and three-dimensional figures.

0016 Understand and apply spatial visualization skills.

For example:

- Translate between two- and three-dimensional representations of geometric figures (e.g., cross sections, nets, projections, perspective drawings).
- Apply procedures for generating solids of revolution from two-dimensional figures.
- Apply techniques of graph theory (e.g., Eulerian and Hamiltonian circuits) to characterize geometric relationships.
- Apply the properties of two- and three-dimensional figures to solve real-world problems.

0017 Understand and apply geometric methods to model mathematical concepts and solve real-world problems.

For example:

- Use coordinate geometry and transformational methods to model mathematical concepts and solve problems involving similarity (including scale and size change), congruence, symmetry, and tessellations.
- Describe and analyze connections among Euclidean, coordinate, and transformational representations of geometric figures.
- Use two- and three-dimensional coordinate systems to represent and analyze geometric figures.
- Describe the relationships between geometry and algebra (e.g., transformations as a geometric equivalence of the function concept).
- Illustrate the applications of recursion and iteration geometrically (e.g., fractals).
- Use a variety of geometric methods (e.g., trigonometric ratios, similarity, proportionality) to solve real-world problems.

SUBAREA V—PROBABILITY AND STATISTICS

0018 Understand counting techniques and the theory of probability.

For example:

- Apply properties of sets and Venn diagrams.
- Determine probabilities in counting situations involving combinations and permutations.
- Find the probability of dependent and independent events.
- Use random variables to interpret and apply probability distributions.
- Analyze problem situations (e.g., fairness of games, lotteries) and determine the probability of events.
- Choose an appropriate simulation to model simple theoretical and experimental probabilities.
- Use probability models and simulations to make and interpret predictions.

0019 Understand the process of posing questions and collecting, organizing, and representing data to answer those questions.

For example:

- Apply criteria for data collection (e.g., random sample, survey techniques).
- Recognize potential sample bias in a given collection technique.
- Calculate and interpret measures of central tendency (mean, median, mode) and variation (e.g., range, standard deviation) to characterize a given set of data.
- Analyze the relationship between data transformations and measures of central tendency and variation.
- Organize and interpret data using a variety of graphs (e.g., bar graphs, line graphs, pictographs, scatter plots, box plots, stem-and-leaf diagrams, histograms, frequency distributions).
- Apply procedures (e.g., geometric, algebraic, calculus) for determining lines of fit and transformations (power and logarithmic transformations) to achieve linearity.

0020 Understand the process of analyzing and interpreting data to make predictions.

For example:

- Draw conclusions about data given summary statistics.
- Identify, analyze, and interpret discrete and continuous data distributions (e.g., binomial, normal distribution).
- Describe the link between probability theory and inferential statistics.
- Identify characteristics of appropriate observations and experiments used in hypothesis testing.
- Compare and contrast concepts of reliability and validity of results.
- Interpret correlation and regression.